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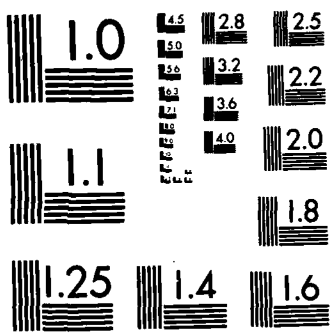

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OPTIMUM PRICING POLICY UNDER STOCHASTIC INFLATION

by

Eytan Sheshinski and Yoram Weiss

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1. Introduction

In this paper we consider pricing policies of individual firms in an inflationary environment. Each firm expects the general price level to increase and must determine the rate of increase of its own price. It is assumed that the firm incurs an adjustment cost when it changes its nominal price. Consequently, firms choose to change prices occasionally rather than continuously.

Our purpose is to analyze the dependence of the magnitude and the frequency of nominal price changes on the inflationary process. This problem has been analyzed by Sheshinski and Weiss [1977] and [1979] for the case of a fixed and certain rate of increase in the aggregate price level. This paper extends the analysis to the case of uncertainty.[1]

The dependence of price policies of individual firms on the aggregate price level implies a relation between relative price dispersion and the inflation rate. This link is an important source of the real costs of inflation as pointed out by Okun [1971]. Extensive empirical research has established the existence of a positive relation between

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the rate of inflation and its variability and relative price dispersion (surveyed recently by Gordon [1981], Fischer [1981] and Taylor [1981]).

We consider an inflationary stochastic process in which the price level changes at intervals of random durations and at a magnitude which is also random. Each firm changes its nominal price whenever its real price falls below some predetermined level, s. The new nominal price is chosen to attain a predetermined real price, S. The duration of the period with fixed nominal price is thus random.

The main result of the paper is that for the class of stochastic processes with an exponential distribution of shock size, the optimal policy is the same as the one obtained under certainty for some specific rate of inflation. This certainty-equivalence rate of inflation always exceeds the expected rate of inflation by a risk premium which depends on the real interest rate and on the parameters of the stochastic process. One can therefore utilize results obtained by Sheshinski-Weiss [1977] for the certainty case to analyze the effects of changes in the parameters of the inflationary process. It is shown that a mean-preserving increase in spread leads to an increase in the amplitude of real price variations and decreases the expected frequency of nominal price changes. A spread-preserving increase in the expected rate of inflation increases the bounds within which real prices vary only if the variability of expected future prices is small. Thus, the main empirical implication of Sheshinski-Weiss [1977] that a higher expected rate of inflation increases the amplitude of real price changes need not hold under more general circumstances.

This corresponds to the empirical findings mentioned in Taylor [1981], suggesting a stronger link between relative price dispersion and the variance of the aggregate inflation rate than with its mean.

2. The Inflationary Process

We describe the inflationary process as a sequence of randomly spaced shocks which are independently and identically distributed. Let p_t be the aggregate nominal price level. Then

$$\log p_t = \sum_{i=1}^{N_t} y_i \tag{1}$$

where N_t is a renewal counting process, i.e. the duration of the periods between successive shocks is i.i.d. with density $q(T)$ (Parzen [1962], Ch. 5). The size of shocks, y_i, is also i.i.d. with density $h(y)$. We further assume that the inflationary process is monotone increasing, i.e. $y_i \geq 0$. This assumption plays an important role in the characterization of the firm's price policy.

For large t, the asymptotic mean, $E(x_t)$, and variance, $Var(x_t)$, of $x_t = \log p_t$ are (Parzen [1962], p. 180)

$$E(x_t) = \frac{E(y)}{E(T)} t \tag{2}$$

and

$$Var(x_t) = \left(\frac{Var(y)}{E(T)} + \frac{Var(T)E(y)^2}{E(T)^3}\right)t \tag{3}$$

where $E(y)$ and $Var(y)$ are the mean and variance of the shock size, y, respectively, and $E(T)$ and $Var(T)$ are the mean and variance of the interarrival time, T. Equations (2) and (3) hold for all t when $q(T)$ is exponential, i.e. when (1) is a Compound Poisson Process.

We may interpret $E(y)/E(T)$, i.e. the product of the average size of shocks and the intensity of shocks as the expected rate of inflation.

We denote by $P_{in}(t - s)$, $t \geq s$, $n \geq i$, $i, n = 0,1,2,\ldots$ the probability of having n shocks by time t conditioned on the i-th shock having occurred at time s. Due to the independence of interarrival times this probability depends only on the difference $t - s$. We further denote by $g_n(p)$, $n = 1,2,\ldots$, the conditional density function of p given that n shocks have occurred. This function is derived from the n-th convolution of $h(y)$, satisfying the recursion[2/]

$$(4) \qquad g_n(p) = \int_0^{\log p} g_{n-1}\left(\frac{p}{e^y}\right) \frac{h(y)}{e^y} dy \quad , \quad n = 2,3,\ldots$$

and

$$g_1(p) = \frac{h(\log p)}{p} \quad .$$

3. The Optimal Pricing Policy

We follow Sheshinski-Weiss [1977] and consider a price-setting firm whose real profits depend on the real price of its product, i.e. the ratio of its own nominal price to the price level. The underlying simplifying assumptions are that at each time consumer's demand is a function of the

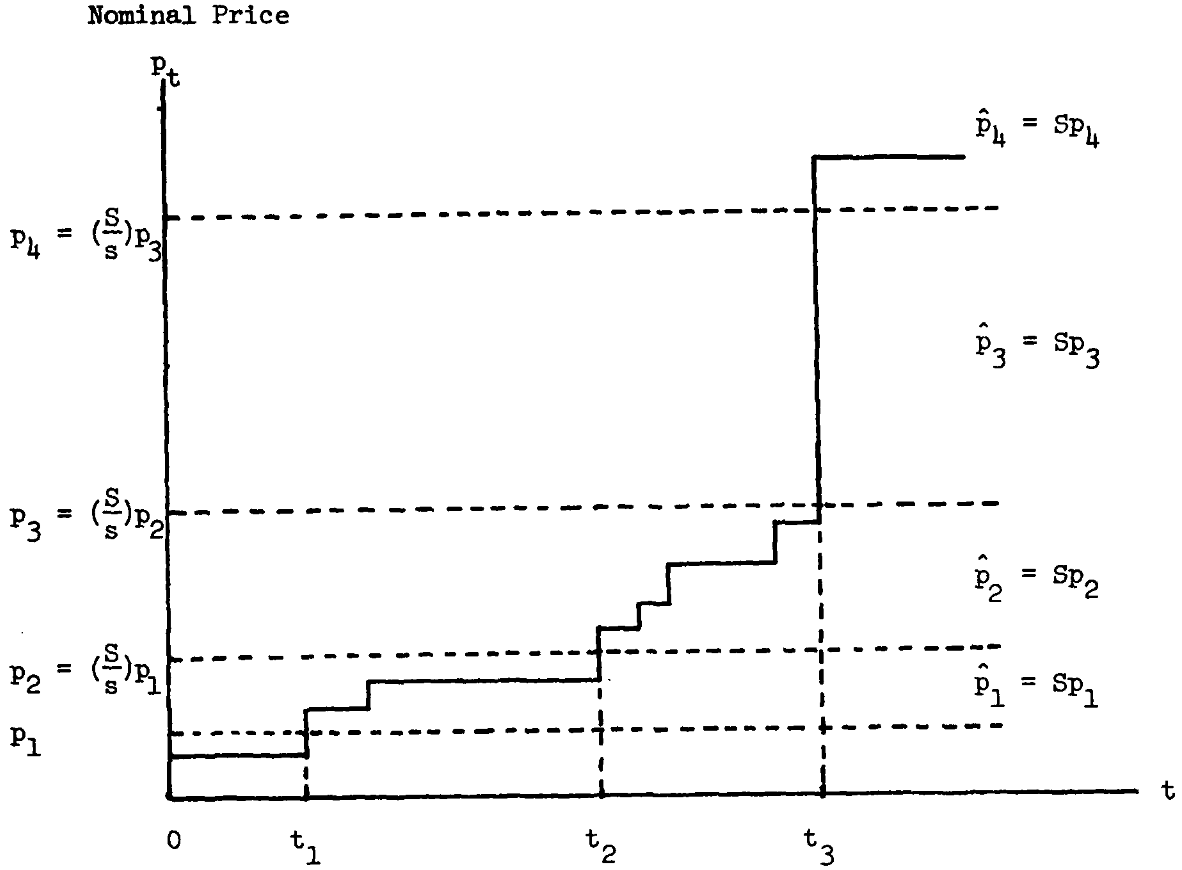
current real price (thus excluding storage and substitution over time). Other factors which may affect demand, such as real incomes, are constant. Similarly, costs adjust instanteneously to the price level.

There are fixed costs associated with nominal price changes and thus the firm will keep its price constant over finite intervals of time. Consequently, real price and real profits are random variables. The distribution of these variables depends on the pricing policy of the firm.

The firm is assumed to be risk neutral. Its objective is to maximize the present value of expected real profits over an infinite horizon. Due to the assumed monotonicity and stationarity of the inflationary process, the problem becomes analogous to the classical inventory problem (Scarf [1959]). In particular, the policy is of the (S,s) form. That is, the firm chooses a critical value s such that whenever its real price falls below (or is equal to) s, it adjusts its nominal price so as to attain a real price of S. It is assumed that the price level is observed instanteneously. Under this policy, the firm changes price only when a shock in the aggregate price level occurs. Thus, the critical price level at which the change occurs is some intermediate level between the price level just prior to the shock and the one just after. It is assumed that in choosing the new nominal price the firm precommits itself to a fixed proportion, S, relative to the above critical price level. (See Figure 1 and a more detailed description in Appendix).

Given this policy, the value of the objective function is determined by the choice of (S,s). We shall first specify the relation between

(a)



$\hat{p}_1, \hat{p}_2, \hat{p}_3, \hat{p}_4, \ldots.$ = Firm's Nominal Prices

$t_1, t_2, t_3, \ldots.$ = Dates of Firm's Price Changes

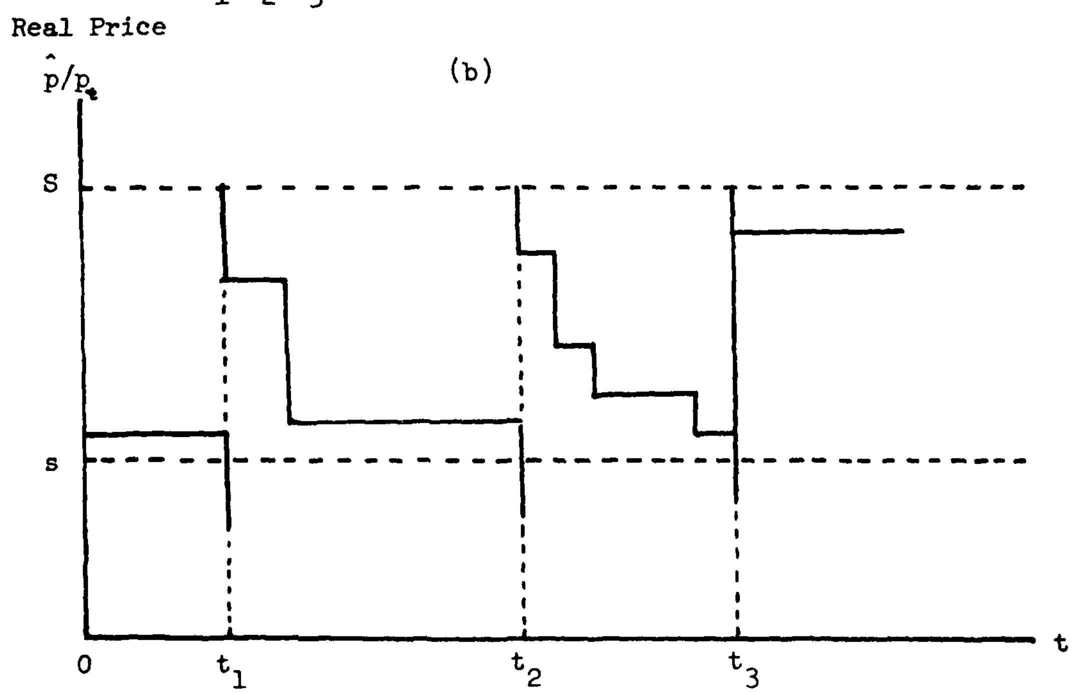


Figure 1

these parameters and the objective function and then characterize their optimal choice.

Consider the firm at (or just prior to) a time of a price change. Due to the assumptions of infinite horizon and the time independence of the interarrival times of the inflationary shocks, the firm's future is independent of the chronological time at which the price change occurs. Thus, without loss of generality, we can reset the price level to be unity and $N_t = 1$ upon any price change.

Let V denote maximal expected discounted real profits net of price adjustment costs, evaluated at a price change. The following recursive formula holds:

$$(5) \qquad V = -\beta + \int_0^\infty e^{-rt} \sum_{n=1}^\infty P_{1n}(t) \int_1^{S/s} F(\tfrac{S}{p})\, g_n(p)\,dp\,dt$$

$$+ V \int_0^\infty e^{-rt} \frac{\partial}{\partial t} \sum_{n=1}^\infty P_{1n}(t) \int_{S/s}^\infty g_n(p)\,dp\,dt + V \int_{S/s}^\infty g_1(p)\,dp$$

where

β = fixed real adjustment costs

r = fixed real interest rate

$F(\cdot)$ = real profits function .

Since we observe the firm at a point of a price change, adjustment costs are subtracted with certainty and without discounting. The first integral on the R.H.S. is the present value of expected profits at time t conditional on the real price being within the range (S,s). The last two

terms on the R.H.S. are the expected present values of the optimal policy following a subsequent price change at time $t \geq 0$.[3/] The term $\partial/\partial t \sum_{n=1}^{\infty} P_{1n}(t) \int_{S/s}^{\infty} g_n(p)dp$ is the density function of $\tau_{S/s}$, the occurrence time of a price change by the firm. This follows from the identity

$$(6) \qquad Pr\{\tau_{S/s} \leq t\} = Pr\{p_t \geq \frac{S}{s}\}$$

$$= \sum_{n=1}^{\infty} P_{1n}(t) \int_{S/s}^{\infty} g_n(p)dp \quad ,$$

which is a consequence of the monotonicity of the inflationary process. The term $\int_{S/s}^{\infty} g_1(p)dp$ is the probability of an additional price change at $t = 0$.[4/]

Integrating the second integral in (5) by parts, and changing the order of integration, we can rewrite (5) as

$$(7) \qquad V = -\beta + \int_{1}^{S/s} F(\frac{S}{p}) L(p)dp + rV \int_{S/s}^{\infty} L(p)dp \quad ,$$

where $L(p)$ is defined as

$$(8) \qquad L(p) = \int_{0}^{\infty} e^{-rt} \sum_{n=1}^{\infty} P_{1n}(t) g_n(p)dt$$

which can be interpreted as the Laplace Transform of the density of p at time t, viewed as a function of t.

4. Certainty Equivalence

Recall from Sheshinski-Weiss [1977] that the recursive formula for V under certainty can be written as

$$(9) \qquad V = -\beta + \int_0^{\varepsilon} F(Se^{-gt})\, e^{-rt}\, dt + e^{-r\varepsilon} V$$

$$= -\beta + \int_1^{S/s} F\left(\frac{S}{p}\right) \frac{1}{g}\, p^{-\frac{r}{g}-1}\, dp + rV \int_{S/s}^{\infty} \frac{1}{g} p^{-\frac{r}{g}-1}\, dp$$

where g is a certain rate of inflation and ε the time-interval between successive price changes.

Let the inflationary stochastic process be specified by a vector of parameters ω and denote the solution to the recursive equation (7) by $V_u(S,s, r,\beta,\omega)$. Similarly, denote the solution to equation (9) by $V_c(S,s,r,\beta,g)$.

Definition 1: A certainty-equivalence rate of inflation g' is a real-valued function $g'(r,\omega)$ such that $V_u(S,s,r,\beta,\omega) = V_c(S,s,r, \beta,g'(r,\omega))$ for all values of S, s, r, β and ω.

Note that we require g' to depend only on parameters of the inflationary process, ω, and on the rate of interest, r. Parameters which vary across firms such as adjustment costs, β, or parameters which characterize the profit function are excluded. This allows us, in analogy to the certainty case, to derive results which are not firm-specific.[5/]

Comparing expressions (7) and (9) the following theorem is immediate:

Theorem 1: A certainty-equivalence rate of inflation g' exists if and only if $L(p)$ can be written in the form

$$L(p) = \frac{1}{g'} p^{-\frac{r}{g'}-1} \tag{10}$$

where g' is a real valued function of the parameters r and ω.

Our objective is to characterize the family of densities of inter-arrival times, $q(T)$, and size of shocks, $h(y)$, which yield the form (10). For this purpose it is useful to note the following property of renewal processes:

Lemma 1

$$\int_0^\infty e^{-rt} P_{1n}(t)dt = \frac{1}{r}(\mathcal{L}_q(r))^{n-1}(1-\mathcal{L}_q(r)) \tag{11}$$

where $\mathcal{L}_q(r) = \int_0^\infty e^{-rt} q(t)dt$ is the Laplace Transform of $q(t)$.

Proof: Let τ_n be the occurrence time of n. Due to the monotonicity of the counting process

$$\Pr\{\tau_n \le t\} = \Pr\{N(t) \ge n\} \quad . \tag{12}$$

Thus,

(13) $$P_{1n}(t) = \Pr\{N(t) \geq n\} - \Pr\{N(t) \geq n+1\}$$
$$= \Pr\{\tau_n \leq t\} - \Pr\{\tau_{n+1} \leq t\}$$

and

(14) $$\frac{dP_{1n}(t)}{dt} = q^*_{n-1}(t) - q^*_n(t) \quad , \quad n = 2,3,\ldots$$

where $q^*_n(t)$ is the n-th convolution of $q(t)$. Integrating by parts and using (14)

(15) $$\int_0^\infty e^{-rt} P_{1n}(t)dt = \frac{1}{r}\int_0^\infty e^{-rt}(q^*_{n-1}(t) - q^*_n(t))dt$$
$$= \frac{1}{r}(\mathcal{L}_q(r))^{n-1}(1 - \mathcal{L}_q(r)) \quad . \quad \text{Q.E.D.}$$

We can now prove the following:

<u>Theorem 2</u>: $L(p) = (1/g')p^{-(r/g')-1}$ if and only if $h(y)$ is exponential.

<u>Proof</u>: We first prove that for all renewal processes, $L(p) = AP^{B-1}$ for some scalars A and B if and only if $h(y)$ is an exponential density.

Let $x = \log p$ and $f^*_n(x) = g_n(e^x)e^x$, $n = 1,2,\ldots$. From (4), f^*_n is the n-th convolution of $h(y)$. By assumption,

(16) $$\int_0^\infty e^{-rt} \sum_{n=1}^\infty P_{1n}(t)f^*_n(x)dt = Ae^{Bx} \quad .$$

Using Lemma 1,

$$Ae^{Bx} = \frac{1}{r}\sum_{n=1}^{\infty} (\mathcal{L}_q(r))^{n-1}(1 - \mathcal{L}_q(r))f_n^*(x) \quad . \tag{17}$$

Taking a Laplace transform on both sides

$$\begin{aligned} \frac{A}{\gamma - B} &= \frac{1}{r}(1 - \mathcal{L}_q(r)) \sum_{n=1}^{\infty} (\mathcal{L}_q(r))^{n-1}(\mathcal{L}_h(\gamma))^n \\ &= \frac{1}{r}\,\frac{(1 - \mathcal{L}_q(r))\mathcal{L}_h(\gamma)}{1 - \mathcal{L}_q(r)\mathcal{L}_h(\gamma)} \end{aligned} \tag{18}$$

where $\mathcal{L}_h(\gamma) = \int_0^{\infty} e^{-\gamma y}h(y)dy$. Solving for $\mathcal{L}_h(\gamma)$,

$$\mathcal{L}_h(\gamma) = \frac{rA/1 - \mathcal{L}_q}{\gamma - (B - \dfrac{rA\mathcal{L}_q}{1 - \mathcal{L}_q})} \quad . \tag{19}$$

Hence,

$$h(y) = \frac{rA}{1 - \mathcal{L}_q}\, e^{(B - \frac{rA\mathcal{L}_q}{1 - \mathcal{L}_q})y} \quad . \tag{20}$$

Since $h(y)$ is known to be a density, we must set

$$\alpha = \frac{rA}{1 - \mathcal{L}_q} = -(B - \frac{rA\mathcal{L}_q}{1 - \mathcal{L}_q}) \quad . \tag{21}$$

Thus,

$$A = \frac{\alpha}{r}(1 - \mathcal{L}_q) \quad , \quad B = \alpha(\mathcal{L}_q - 1) \quad . \tag{22}$$

Define

$$g' = \frac{r}{\alpha(1 - \mathcal{L}_q(r))} \quad . \tag{23}$$

It is now easily verified that $A = 1/g'$ and $B = -r/g'$ as required in the statement of the theorem∎.

Definition (23) provides a constructive method to calculate the certainty-equivalence rate of inflation. It is seen to depend on r, α and on the parameters of the density $q(T)$. A natural question is the relation between this rate and the expected rate of inflation, $E(y)/E(T)$. The following definition and proposition address this issue.

<u>Definition 2</u>: $R(r,\omega) = g'(r,\omega) - E(y)/E(T)$ is the <u>risk-premium</u> associated with a rate of interest r and a stochastic inflationary process parametrized by ω.

<u>Proposition 1</u>: $R = 0$ when $r = 0$ and $dR/dr > 0$.

<u>Proof</u>: Expanding $\mathcal{L}_q(r)$ by a Taylor's series around $r = 0$,

$$1 - \mathcal{L}_q(r) = E(T)r - \frac{E(T^2)}{2}r^2 + \ldots \tag{24}$$

and using $E(y) = 1/\alpha$,

$$R = E(y)\left(\frac{1}{E(T) - \frac{E(T^2)}{2}r + \frac{E(T^3)}{3!}r^2 - \ldots} - \frac{1}{E(T)}\right) \geq 0 \tag{25}$$

Thus, $R = 0$ when $r = 0$. Furthermore, for $r > 0$

$$(26)\qquad \frac{dR}{dr} = \frac{dg'}{dr} = \frac{1}{\alpha(1 - \mathcal{L}_q(r))^2}\left(1 - \mathcal{L}_q(r) + r\,\frac{d\mathcal{L}_q(r)}{dr}\right)$$

$$= \frac{1}{\alpha(1 - \mathcal{L}_q)^2}\int_0^\infty (e^{rt} - 1 - rt)e^{-rt}q(t)dt > 0 \quad \blacksquare.$$

It should be noted that when a certainty equivalence exists, the optimal (S,s) values chosen under uncertainty converge to their values under certainty (and $V_c - V_u$ tends to zero) as r approaches zero. The reason is that for given (S,s), different time-paths of the real price have the same effect on real profits. In addition, the expected frequency of price changes will be the same. When $r > 0$, the timing of the realization of various real prices is relevant.

The precise form of $g'(r,\omega)$ depends on the specification of the stochastic process. The density of shock size, $h(y)$, has been restricted by Theorem 1, to be $h(y) = \alpha e^{-\alpha y}$. If we further specify the density of interarrival times, $q(T)$, to be also exponential, say $q(t) = \lambda e^{-\lambda t}$, then g' assumes the simple form

$$(27)\qquad g' = \frac{\lambda + r}{\alpha} \quad .$$

In this case, the characterization of the inflationary process $x_t = \log p_t$, given in (2) and (3) becomes

(28) $$E(x_t) = \frac{\lambda}{\alpha} t$$

and

(29) $$\mathrm{Var}(x_t) = \frac{2\lambda}{\alpha^2} t$$

for all t. The risk-premium thus becomes

(30) $$R = \frac{r}{\alpha} \ .$$

The certainty case can be obtained by letting $\alpha, \lambda \to \infty$ while holding the ratio λ/α constant. In the limit, the variance and the risk-premium approach zero.

5. Comparative Statics

In this section we wish to investigate the effect of changes in the parameters of the inflationary process on the choice of S and s. The implications for the expected time between successive price adjustments by the firm will be discussed in the following section.

Since the compound Poisson is a renewal process which admits simple interpretation of the parameters in terms of the observed mean and variance of prices we shall restrict ourselves to this case. We shall assume that a certainty-equivalence exists, i.e. that $h(y)$ is exponential. For this case, the results in Sheshinski-Weiss [1977] are directly applicable. It has been shown there that under certainty an increase in the rate of inflation increases S, decreases s and hence increases the amplitude

of real price variations. The effects of the parameters of the inflationary process on the optimum (S,s) can therefore be inferred from their effect on the certainty-equivalence rate $g'(r,\alpha,\lambda)$.

An increase in the intensity of price shocks, λ, is seen to increase g' and hence will increase S and reduce s. An increase in the expected shock size, $1/\alpha$, has a similar effect. Each of these cases involves simultaneous changes in the mean and variance of the price level. It is, however, possible to separate these effects. In view of (28) and (29), a mean-preserving increase in the variance of the price level is attained by reducing α and λ at the same rate. As seen from (27), such a change leads to an increase in g'. We thus draw the important conclusion that an increase in the variability of the price level unambiguously leads to an increase in the amplitude of the firm's real price.

Spread preserving changes may lead to ambiguous results depending on the measure of spread. Increasing the mean holding the variance constant will increase g' only if $\lambda/(\lambda + r) > 1/2$ which holds for large λ, i.e. when uncertainty is small. However, holding the coefficient of variation constant, which requires a constant λ, and increasing the mean will lead to larger S and smaller s, as in the certainty case.

6. Expected Frequency of Price Changes

Another variable of interest is the time between successive price changes by the firm. In contrast to the certainty case, under uncertain inflation, this is a random variable. We shall limit the analysis to the effect of the parameters of the inflationary process on the expected time between successive price changes.

Let $\tau_{S/s}$ be the earliest time at which $p_t = S/s$, given $p_0 = 1$. Since all shocks are non-negative, we have the identity

$$(31) \qquad \Pr\{\tau_{S/s} \leq t\} = \Pr\{p_t \geq \frac{S}{s}\}$$

$$= \sum_{n=1}^{\infty} P_{1n}(t) \int_{S/s}^{\infty} g_n(p)dp \quad ,$$

and hence

$$(32) \qquad E(\tau_{S/s}) = \int_0^{\infty}(1 - \sum_{n=1}^{\infty} P_{1n}(t) \int_{S/s}^{\infty} g_n(p)dp)dt$$

$$= \int_0^{\infty} \sum_{n=1}^{\infty} P_{1n}(t) \int_1^{S/s} g_n(p)dp\,dt \quad .$$

$$= E(T) \sum_{n=1}^{\infty} G_n(\frac{S}{s})$$

where $G_n(S/s) = \int_1^{S/s} g_n(p)dp.$ 6/ Expected waiting-time is thus seen to be equal to the product of the expected time between shocks and the expected number of shocks with an associated price in the range $(1,S/s)$, which is 'Wald's Identity' (Feller [1971], p. 397).

If we assume that a certainty-equivalence exists, i.e. $h(y) = \alpha e^{-\alpha y}$, and that $q(T)$ is also exponential, i.e. $q(t) = \lambda e^{-\lambda t}$, then (32) simplifies to 7/

$$(33) \qquad E(\tau_{S/s}) = \frac{\alpha}{\lambda} \log \frac{S}{s} \ .$$

Recalling that, in this case, λ/α is the expected rate of inflation, the relation between the expected values in (33) is identical to the relation obtained under certainty (Sheshinski-Weiss [1977]).

As in the certainty case, the relation between the expected rate of inflation and $E(\tau_{S/s})$ is ambiguous. However, we get the important result that an increase in the variability of inflation, holding the mean constant, unambiguously reduces the expected frequency of price changes. This follows directly from our previous remarks that such a change increases S and decreases s.

7. Interactions Across Firms

So far we have examined the behavior of a representative firm in isolation. The inflationary process was treated as exogeneous rather than an outcome of the actions of individual firms. Nor did we discuss the source of the aggregate disturbances. A detailed analysis of these issues is beyond the scope of this paper. Nevertheless, we would like to outline a possible framework for such an analysis.

Suppose the economy consists of two sectors: a competitive sector of price-takers and a monopolistic sector of price setters. Assume that exogeneous shocks in costs or in demand affect the competitive sector. Since competitive firms cannot adopt independent price policies, this sector will adjust its price immediately. The resulting change in relative prices will induce a change in the demand facing firms in the monopolistic sector.

In this sector price changes in general do not occur immediately. On average, however, monopolistic firms adjust their price at the same rate as competitive firms. The aggregate outcome is therefore consistent with the expectations of each firm. Whenever a firm changes its nominal price, it increases it by the rate of $\log (S/s)$ and keeps it constant for an average duration of $E(\tau_{S/s})$. As seen from (33), optimal behavior implies that the rate of price change per unit time of each firm is on average equal to the expected rate of inflation.

A stronger test of consistency would require fulfillment of expectations at any moment in time and not only on average. Under certainty, conditions for such consistency can be easily described. If the dates of price changes by firms are uniformly distributed, then the aggregate price level will increase continuously at a constant rate, as expected by firms, even though every firm in isolation follows a discontinuous price policy. Under uncertainty we can no longer assume that the distribution of the dates of price changes is invariant over time. If all firms are identical in their (S,s), then price changes will be eventually fully synchronized, yielding discontinuous aggregate price behavior as expected by each firm. This is due to the positive probability for sufficiently large shocks which will induce all firms to change their prices.

Appendix

The purpose of this appendix is to prove the uniqueness and optimality of the (S,s) policy when a certainty equivalence exists.

The (S,s) policy stipulated in the text is a mixture of <u>ex-ante</u> and <u>ex-post</u> decisions. That is, price changes are undertaken depending upon the realization of a shock while the nominal price change is predetermined, independently of the size of the last shock.

Alternatively, (S,s) policies can be formulated in a purely <u>ex-post</u> or <u>ex-ante</u> fashion. In the <u>ex-post</u> policy the firm's nominal price is adjusted so as to attain a fixed real price (S) in terms of the price level immediately <u>after</u> the realization of a shock. Similarly, the criterion for undertaking a price change is the real price following a shock, i.e. the ratio of the firm's nominal price prior to a change to the nominal price level after the realization of a shock. Analogously, for the <u>ex-ante</u> (S,s) policy, the critical values are defined in terms of the ratios of the firm's old and new nominal prices to the nominal price level just prior to a shock.

Using the conditions proposed by Scarf [1959], it is easy to establish the optimality of (S,s) policies in the ex-ante and ex-post cases. The choice between the alternative (S,s) policies depends upon whether the firm must precommit itself in announcing prices. Some precommitment seems necessary if changes in the nominal price level are to reflect the actions of individual firms.

Neither the <u>ex-post</u> nor the <u>ex-ante</u> (S,s) policies yield a certainty equivalence. The reason is that contrary to the certainty case the firm spends finite amounts of time at S (and infinitesimal amounts of time at s). The mixed policy described in the text yields a more symmetric pattern. However, its optimality can be established only for and exponential distribution of shocks.

Consider the firm at a point in time, t_0, when a shock occurs. Let the nominal price level be p_0 and the firm's nominal price be $z_0 p_0$ just prior to t_0. Thus, z_0 is the real price at t_0. Let $V(z_0)$ be the maximized discounted expected profits as a function of the real price. Due to the infinite horizon and the stationarity of the stochastic inflationary process, V is independent of t_0.

We first wish to provide a sufficient condition for price changes to occur only at shocks.

<u>Theorem 1A:</u> If the density of waiting-time to the next shock, conditioned on no shock having occured up to time x, $q_x(t) = \frac{q(t + x)}{1 - Q(x)}$, is a monotone non-increasing function of x then it is never optimal to change prices between shocks.[8/]

<u>Proof</u>: Suppose a shock of size y occured at time t_0 and let t_1, $t_1 > t_0$, be a time such that $N(t_1) = N(t_0) + 1$ (i.e. no shock has occured in $(t_0,t_1])$. Set the nominal price level after the shock to unity and let z_1 be the firm's nominal (and real) price just after t_0. The firm considers a price change at t_1 to $z_1^* > z_1$. This change is profitable if

$$\text{(1a)} \qquad \int_0^\infty (F(z_1^*)\frac{1}{r}(1 - e^{-rt}) + V(z_1^*)e^{-rt})q_{t_1}(t)dt - \beta >$$

$$> \int_0^\infty (F(z_1)\frac{1}{r}(1 - e^{-rt}) + V(z_1)e^{-rt})q_{t_1}(t)dt \quad .$$

Now, take $t_2 \in (t_0,t_1)$. Due to the monotonicity assumption $q_{t_2}(t) \geq q_{t_1}(t)$, $\forall t \geq 0$. Thus, replacing $q_{t_1}(t)$ with $q_{t_2}(t)$ in (1a) retains the inequality. Hence it would be profitable for the firm to choose the same nominal (and real) price as in t_1 and hold it fixed until the next shock including t_1. By a similar argument, if the firm contemplates additional changes in its nominal price between t_1 and the next shock, one can show that it is profitable to choose the same price as t_1 somewhat earlier and to follow the same pattern of price changes thereafter. This includes not changing the price of t_1. An optimal price choice at t_2 can only increase this gain. It follows that a price change cannot occur between shocks∥.

Since price changes are costly, the firm will avoid price adjustments in the absence of a change in the current or expected aggregate price level. In the absence of shocks the current price level is unchanged and the expectation for a price shock decreases due to the monotonicity assumption. In these circumstances, additional information only decreases the incentive for a price change. Note that the Poisson process, $q(t) = \lambda e^{-\lambda t}$, and more generally the Gamma density, $q(t) = \frac{\lambda^{\alpha} t^{\alpha-1}}{\Gamma(\alpha)} e^{-\lambda t}$, with $0 < \alpha \leq 1$ (see Barlow and Proschan [1975, ch.3]), satisfy the condition of Theorem 1A.

The next Theorem provides sufficient conditions for the optimality of (S,s) policies for the mixed ex-post, ex-ante case analysed in the text. It will be assumed that the sufficient condition for the optimality of pure ex-ante (S,s) policies, i.e. β-concavity of the profit function $F(\cdot)$, is satisfied.

Theorem 2A. If $h(y)$ is exponential then the optimal policy is (S,s).

Proof. Due to Theorem 1A and given our assumption of precommitment the value function defined at points of shock satisfies the following recursive equation

$$(2a) \qquad V(z_0) = \underset{z_1}{\text{Max}} \, [\int_0^\infty \text{Max} \, \{\int_0^\infty (F(z_0 e^{-y})\frac{1}{r}(1 - e^{-rt}) + V(z_0 e^{-y})e^{-rt})q(t)dt,$$

$$- \beta + \int_0^\infty (F(z_1 e^{-y})\frac{1}{r}(1 - e^{-rt}) + V(z_1 e^{-y})e^{-rt})q(t)dt\}h(y)dy] \quad .$$

Given a real price z_0 when a shock occurs, the firm decides on a strategy for every possible realization of the size of the immediate shock, taking into account the effects of its policy until the arrival of the next shock. The firm is assumed to precommit itself to a nominal price $z_1 p_0$ but it can choose whether to change the price to $z_1 p_0$ or to keep the current nominal price, $z_0 p_0$, depending on the realization of the immediate shock.

Denote by $\psi(z)$ the function

$$\psi(z) = \int_0^\infty (F(z)\frac{1}{r}(1 - e^{-rt}) + V(z)e^{-rt})q(t)dt \quad . \tag{3a}$$

Then the F.O.C. for maximization of the R.H.S. of (2a) are

$$\psi(z_0 e^{-y^*}) - \psi(z_1 e^{-y^*}) + \beta = 0 \quad , \tag{4a}$$

and

$$\int_{y^*}^\infty \psi'(z_1 e^{-y})h(y)e^{-y}dy = 0 \quad . \tag{5a}$$

where it is assumed that the solution y^* to (4a) is unique for any z_0 and z_1. A sufficient condition for the uniqueness of y^* is that $F(z)$ and thus $V(z)$ and $\psi(z)$, are <u>β-concave</u> (see Scarf [1959]), i.e.

$$-\beta + F(z + a) - F(z) - aF'(z) \leq 0 \qquad \forall a,z \geq 0 \quad , \tag{6a}$$

and given $\beta \geq 0$. Let us denote

$$S = z_1 e^{-y^*} \quad \text{and} \quad s = z_0 e^{-y^*} \quad . \tag{7a}$$

Assume further that the density $h(y)$ is exponential. Then conditions (4a) - (5a) can be rewritten:

$$\psi(s) - \psi(S) + \beta = 0 \tag{8a}$$

$$\int_0^\infty \psi'(Se^{-y})h(y)e^{-y}dy = 0 \tag{9a}$$

which yields values (S,s) independent of the initial conditions z_0.

The optimal policy can thus be described as follows. If the firm finds itself with $z_0 \leq s$ it changes its price immediately to S, without waiting for the realization of the shock. If after the shock the firm's real price, z_0e^{-y} falls below s the firm also adjusts its price to the same value S. If during the shock the firm's real price is above s, the firm will not change its price. Under the exponential shock distribution we can further assume that the maximation problem (2a) is solved for <u>any</u> value of the real price during a shock. This, in particular, if $z_1e^{-y} < s$, a new nominal price is chosen and the problem is solved again.

Substituting $V(s)$ for V in (5), one can verify that the recursive relation (5) holds. At any point in time in which the firm changes its price (which must be a point of a shock) we define a critical price level which induces the price change. If one sets this value to unity then the next price change by the firm will occur when the price level reaches S/s. Furthermore, due to the assumption that $h(y)$ is exponential, the conditional distribution of the normalized p satisfies (4). This can be shown as follows. At any shock where the firm changes its price, the critical price level is $p_0e^{y^*}$, where y^* is determined by conditions (4a)-(5a). Thus, the normalized price level, following the price change, $p = p_0e^y/p_0e^{y^*} = e^{y-y^*}$, is distributed according to

$$\Pr\{P \leq p\} = \Pr\{y - y^* \leq \log p | y - y^* \geq 0\} \quad . \tag{10a}$$

Under the exponential distribution for y, this yields a density for p

$$g_1(p) = \frac{h(\log p)}{p} \tag{11a}$$

as in (4).

Having established that for the certainty-equivalence case the optimal policy is (S,s), we shall now show its uniqueness.

The maximized value of V can be expressed in terms of S and s and will satisfy the recursive equation (7), which implies

$$V(S,s) = \frac{-\beta + \int_1^{S/s} F(\frac{S}{p})\, L(p)dp}{1 - r \int_{S/s}^{\infty} L(p)dp} \tag{12a}$$

$$= \frac{1}{r} \frac{-\beta + \int_1^{S/s} F(\frac{S}{p})\, L(p)dp}{\int_1^{S/s} L(p)dp}$$

having used $\int_0^{\infty} L(p)dp = 1/r$.

The first-order conditions for an interior maximum are

$$F(s) - rV = 0 \tag{13a}$$

$$\int_1^{S/s} F'(\frac{S}{p}) \frac{L(p)}{p}\, dp = 0 \quad . \tag{14a}$$

The second-order conditions for a local maximum are equivalent to $F'(s) > 0$ and

(15a) $$\begin{vmatrix} -F'(s) & 0 \\ 0 & F'(S)L(1) + \int_1^{S/s} F'(\frac{S}{p})L'(p)dp \end{vmatrix} > 0 \quad ,$$

where $\partial^2 V/\partial S^2 = 1/S(F'(s)L(S/s) + S\int_1^{S/s} F''(S/p)(L(p)/p^2)dp)$ has been integrated by parts to obtain the second-diagonal element in the determinant and all derivatives are evaluated at a point which satisfies (13a)-(14a). Unlike the certainty case, the sign of the last term cannot be determined without some restrictions on the stochastic process.

Quasi-concavity, implied by the concavity of $F(\cdot)$, together with conditions (13a) and (14a) yields that $F'(S) < 0$. Then, in addition, the following condition can be shown to be sufficient for $\partial^2 V/\partial S^2 < 0$:

(16a) $$\frac{d}{dp}\left(\frac{L'(p)p}{L(p)}\right) \leq 0 \quad .$$

The proof is immediate upon rewriting

$$\int_1^{S/s} F'(\frac{S}{p})L'(p)dp = \int_1^{S/s} \left(F'(\frac{S}{p})\frac{L(p)}{p}\right)\left(\frac{L'(p)p}{L(p)}\right)dp$$

and integrating by parts, using the first-order condition (14a).

This sufficient condition is satisfied when a certainty equivalence exists (and in the case of certainty), since by Theorem 1, $L(p)$ has in this case a constant elasticity.

In the absence of a condition such as (16a), S may not be unique and discontinuity w.r.t. the parameters may arise. Note, however, that concavity of $F(\cdot)$ and condition (13a) imply that s is unique.

Footnotes

1/ Extensions of Sheshinski-Weiss [1977] model to the case of uncertainty have been made in unpublished works by Padan [1981], Danziger [1981] and Roberds [1979]. The present work has benefited from our direct interaction with Padan and Danziger.

2/ Let

$$G_n(p) = \Pr\{P \leq p \text{ after } n \text{ shocks}\}$$
$$= \int_0^{\log p} \Pr\{P \leq \frac{p}{e^y} \text{ after } n-1 \text{ shocks}\} h(y)dy$$
$$= \int_0^{\log p} G_{n-1}(\frac{p}{e^y})h(y)dy \quad .$$

The density $g_n(p)$ is given by $G_n'(p)$, using $G_n(1) = 0$.

3/ The conditional density of p in (5) depends in general on the critical price level which one uses for normalization. Only for $h(y)$ exponential is $g_1(p)$ (and thus $g_n(p), n = 2,3,\ldots$) independent of the normalization. Equation (5) holds, however, for any distribution of shock size if the normalization is in terms of the price level just prior to the shock.

4/ The assumption of precommitment is reflected in recursion (5) by the variation in the <u>real</u> price following a nominal price change. If the firm has no precommitment, it will choose a price strategy, with random nominal price, so as to attain a predetermined real price. In this case (5) becomes

$$(5') \quad V = -\beta + \int_0^\infty e^{-rt} \sum_{n=2}^{\infty} P_{1n}(t) \int_1^{S/s} F(\frac{S}{p})g_n(p)dp$$

$$+ V\int_0^\infty e^{-rt} \frac{\partial}{\partial t} \sum_{n=2}^{\infty} P_{1n}(t) \int_{S/s}^{\infty} g_n(p)dp + F(S)\int_0^\infty e^{-rt}P_{11}(t)dt \quad .$$

The difference between (5) and (5') is that in the absence of further shocks the price distribution is degenerate. Specifically,

$$G_1(p) = \begin{cases} 0 & p < 1 \\ 1 & p \geq 1 \end{cases} .$$

This means that the real price remains at S until a further shock occurs.

5/ Definition 1 is equivalent to the requirement that S and s will be the same functions of g under certainty and g' under uncertainty. This is necessary for the equivalence of the comparative statics. A weaker definition will equate the value of the optimal solution (evaluated, possibly, at different (S,s) values).

6/ By (13), $P_{1n}(t) = Q_n(t) - Q_{n+1}(t)$, where $Q_n(t) = P_r\{\tau_n \leq t\}$. Hence,

$$\int_0^\infty P_{1n}(t)dt = \int_0^\infty (1 - Q_{n+1}(t))dt - \int_0^\infty (1 - Q_n(t))dt$$

$$= E(\tau_{n+1}) - E(\tau_n) = E(T) .$$

7/ For this case $g_n(p) = \alpha \frac{(\alpha \log p)^{n-1}}{(n-1)!} p^{-\alpha-1}$. Hence,

$$\sum_{n=1}^\infty \int_1^{S/s} g_n(p)dp = \int_1^{S/s} \alpha p^{-\alpha-1} \sum_{n=0}^\infty \frac{(\alpha \log p)^n}{n!} dp$$

$$= \alpha \log (\frac{S}{s}) .$$

8/ This monotonicity assumption is equivalent to the condition of decreasing hazard-rate, $\frac{q(t)}{1 - Q(t)}$ $(Q(t) = \int_0^t q(x)dx)$. For a discussion of this issue see Barlow and Proschan [1975, Ch.3].

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